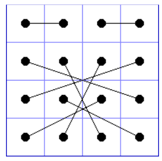
Theorem 7 [connection figure (1122 3456 6783 8547)]



(i) Let k,r be natural numbers with either

(A) 1 < k < r+1, (B) r+1 < k < 2r+1,
(C) 2r+1 < k < 4r+1, (D) 4r+1 < k < 6r+1
(E) 6r+1 < k < 8r+1, (F) 8r+1 < k < 9r+1,
(G) 9r+1 < k < 10r+1, (H) 10r+1 < k.

Then the 16 numbers

(*) 1,r+1,2r+1,4r+1,6r+1,8r+1,9r+1,10r+1,
k,k+r,k+2r,k+4r,k+6r,k+8r,k+9r,k+10r

are pairwise different and (*) is a symmetric subset of {1,...,N}, N=k+10r, with difference set, casewise:

(A) k-1,r-k+1,k-1,r-k+1,k-1,2r-k+1,k-1,2r-k+1,k-1,2r-k+1,k-1,r-k+1,k-1,r-k+1,k-1

(B) r,k-r-1,2r-k+1,k-r-1,r,2r-k+1,k-1,2r-k+1,k-1,2r-k+1,r,k-r-1,2r-k+1,k-r-1,r

(C) r,r,k-2r-1,r,3r-k+1,k-2r-1,4r-k+1,k-2r-1,4r-k+1,k-2r-1,3r-k+1,r,k-2r-1,r,r

(D) r,r,2r,k-4r-1,r,5r-k+1,k-4r-1,6r-k+1,4r-k-1,5r-k+1,r,k-4r+1,2r,r,r

(E) r,r,2r,2r,k-6r-1,r,7r-k+1,k-6r-1,7r-k+1,r,k-6r-1,2r,2r,r,r

(F) r,r,2r,2r,2r,k-8r-1,9r-k+1,k-8r-1,9r-k+1,k-8r-1,2r,2r,2r,r,r

(G) r,r,2r,2r,2r,r,k-9r-1,10r-k+1,k-9r-1,r,2r,2r,2r,r,r

(H) r,r,2r,2r,2r,r,r,k-10r-1,r,r,2r,2r,2r,r,r

Moreover, there are 4 different general 4x4 magic squares M1, M2, M3, M4 with entries from (*) and connection figure (1122 3456 6783 8547), namely

M1 =

k	10r+1	1	k+10r
k+9r	4r+1	6r+1	k+r
9r+1	k+2r	k+8r	r+1 ,
2r+1	k+4r	k+6r	8r+1

M2 =

1	k+10r	k	10r+1
9r+1	k+4r	k+6r	r+1
k+9r	2r+1	8r+1	k+r .
k+2r	4r+1	6r+1	k+8r

M3 and M4, are obtained from M1 and M2 by vertical reflection.

(ii) Every general magic square M with entries from a symmetric subset of {1,...,N}, with connection figure (1122 3456 6783 8547), and with entry 1, is of the form either M1,M2,M3,M4, and the corresponding difference vector of this subset is of the form either A,B,C,D,E,F,G, or H.

Proof

(i) can be verified by a simple calculation,
(ii) can be proved by solving the linear equations for M.

Remark

For each symmetric subset of {1,...,N} containing the number 1, and allowing a general 4x4 magic square of connection figure (1122 3456 6783 8547)there are exactly 4 different general 4x4 magic squares with entries from this subset. There exists an imbedding from the set of general 4x4 magic squares of connection figure (1122 3456 6783 8547) into the set of general 4x4 magic squares of connection figure (1234 2567 8653 4871) [see Theorem 11, Remark 2 (ii)].