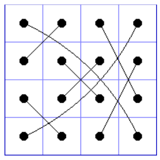
Theorem 11 [connection figure (1234 2567 8653 4871)]



(i) Let k,r,s be natural numbers with s<r and

```
(I)   2s<r and either
(AI)  1<k<s+1             (BI)  s+1<k<2s+1
(CI)  2s+1<k<r+1          (DI)  r+1<k<r+2s+1
(EI)  r+2s+1<k<2r+1       (FI)  2r+1<k<2r+s+1
(GI)  2r+s+1<k<2r+2s+1    (HI)  2r+2s+1<k,

or

(II)  r<2s and either
(AII) 1<k<s+1          (BII) s+1<k<r+1          (CII) r+1<k<2s+1          (DII) 2s+1<k<2r+1
(EII) 2r+1<k<r+2s+1    (FII) r+2s+1<k<2r+s+1    (GII) 2r+s+1<k<2r+2s+1    (HII) 2r+2s+1<k.
```

Then the 16 numbers

```
(*)  1,1+s,1+2s,1+r,1+r+2s,1+2r,1+2r+s,1+2r+2s,
     k,k+s,k+2s,k+r,k+r+2s,k+2r,k+2r+s,k+2r+2s
```

are pairwise different and (*) is a symmetric subset of {1,...,N}, N=k+2r+2s.
Moreover, there are 16 different general 4x4 magic squares M01, M02, ..., M16
with entries from (*) and connection figure (1234 2567 8653 4871), namely

```
      1+s      1+2r     k+2r+2s  k+s
      k+2s     k+r      1+r      1+2r+2s
M01 = k+2r     k+r+2s   1+r+2s   1
      1+2r+s   1+2s     k        k+2r+s

      1+s      1+2r+2s  k+2r     k+s
      k        k+r+2s   1+r+2s   1+2r     (in M01 the [virtual] summand 0s has
M02 = k+2r+2s  k+r      1+r      1+2s      been exchanged with 2s)
      1+2r+s   1        k+2s     k+2r+s
```

M03 and M04 are derived from M01 and M02 by exchanging 1 and k,
M05,M06,M07, and M08 arise from M01,M02,M03 and M04 by exchange of r and s, and
finally, M08,M09,...,M16 are the mirror images of M01,M02,...,M08 by horizontal
reflection.

(ii) Every general 4x4 magic square M with entries from a symmetric subset of {1,...,N}, with connection figure (1234 2567 8653 4871), and with entry 1, is of the form either M1,M2,...,M15, or M16; and the corresponding parameters k,r,s for this subset fulfil the inequalities either AI,BI,CI,DI,EI,FI,GI,HI,AII,BII,CII,DII,EII,FII,GII, or HII.

Proof

(i) is verified easily; (ii) follows, when the linear equations for M are solved.

Remark 1

For each symmetric subset of {1,...,N} containing the number 1, and
allowing a general 4x4 magic square of connection figure (1234 2567 8653 4871)
there are exactly 16 different general 4x4 magic squares with entries from
this subset.
The smallest N, with a symmetric subset allowing a general 4x4 magic square with
connection figure (1234 2567 8653 4871) and 1 as an entry, is N=18.
From N=18 to N=29 every case AI,...,HII occurs.

Remark 2

(i) With s=3r in (*) one has the 16 numbers of (*) from Theorem 06. The map

```
c01 c02 c03 c04       c06 c02 c10 c14
c05 c06 c07 c08       c01 c05 c13 c09
c09 c10 c11 c12  -->  c04 c08 c16 c12
c13 c14 c15 c16       c07 c03 c11 c15
```

brings any general magic 4x4 square of connection figure (1122 3456 7878 6543) onto a
general 4x4 magic square of connection figure (1234 2567 8653 4871).

(ii) With s=4r in (*) one has the 16 numbers of (*) from Theorem 07. The map

```
c01 c02 c03 c04       c06 c13 c04 c14
c05 c06 c07 c08       c11 c08 c12 c02
c09 c10 c11 c12  -->  c10 c05 c09 c03
c13 c14 c15 c16       c07 c16 c01 c15
```

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brings any general magic 4x4 square of connection figure (1122 3456 6783 8547) onto a general 4x4 magic square of connection figure (1234 2567 8653 4871)

(iii) There is an imbedding map i of the set of GMS of connection figure (1234 2567 8653 4871) into the set of GMS of connection figure (1122 3443 5665 7788), defined by:

```
    c01 c02 c03 c04       c12 c03 c14 c09
    c05 c06 c07 c08       c06 c13 c04 c11
i:  c09 c10 c11 c12  -->  c10 c01 c16 c07.
    c13 c14 c15 c16       c08 c15 c02 c05
```

This follows from seven additional equations, valid for every general magic square of connection figure (1234 2567 8653 4871):

c01+c03+c10+c12 = 2(N+1), c01+c04+c08+c09 = 2(N+1), c04+c06+c11+c13 = 2(N+1),
c02+c04+c14+c16 = 2(N+1), c05+c07+c09+c11 = 2(N+1), c05+c12+c13+c14 = 2(N+1),
c06+c08+c10+c12 = 2(N+1).