Sketch of the proof for the existence of exactly 145 connection figures

Consider a list of strings with 16 entries from {1,…,8}, beginning with 1 and every entry occurring twice; the next free first of the 2 equal numbers always is put onto the next free place. There are 2,027,025 of such strings.
Example: (1122 3434 5566 7887).
Read every string as 4x4 matrix and imagine equal entries of the matrix connected by a line. Delete from the list of strings every string, n o t fulfilling the condition:
“if two entries in a row, column or diagonal are connected, then the other two entries of this row, column or diagonal are connected, too”.

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                                      # . . #    . . . .    . # # .        . . . .
                                      . . . .    . # # .    . . . .        # . . #
Then the same is done for the blocks  . . . .,   . # # .,   . . . .,  and  # . . #,
                                      # . . #    . . . .    . # # .        . . . .
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If 2 entries from one of these blocks are connected, then the other two entries from this block must be also connected.

Now every string from the list is deleted, which is the image of a string, earlier in the list, under a rotation or reflection.

The above example belongs to the remaining strings.

Now for every of these strings a system of linear equations for a general 4x4 magic square with the connection figure in question is established:

sum of row no. i = sum of row no. 1, i=2,3,4
sum of column no. i = sum of row no. 1, i=1,2,3,4
sum of diagonal no. i = sum of row no. 1, i=1,2

8 equations have to be added: when entry x_i and entry x_j are connected, then the additional corresponding equation is: $2x_i + 2x_j$ = sum of row no. 1.

Example For the string (1122 3434 5566 7887) the linear equation system is Ax=b with vector b=(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0) and matrix A:

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[-1,-1,-1,-1, 1,1,1,1, 0,0,0,0, 0,0,0,0]
[-1,-1,-1,-1, 0,0,0,0, 1,1,1,1, 0,0,0,0]
[-1,-1,-1,-1, 0,0,0,0, 0,0,0,0, 1,1,1,1]

[ 0,-1,-1,-1, 1,0,0,0, 1,0,0,0, 1,0,0,0]
[-1, 0,-1,-1, 0,1,0,0, 0,1,0,0, 0,1,0,0]
[-1,-1, 0,-1, 0,0,1,0, 0,0,1,0, 0,0,1,0]
[-1,-1,-1, 0, 0,0,0,1, 0,0,0,1, 0,0,0,1]

[ 0,-1,-1,-1, 0,1,0,0, 0,0,1,0, 0,0,0,1]
[-1,-1,-1, 0, 0,0,1,0, 0,1,0,0, 1,0,0,0]

[ 1, 1,-1,-1, 0,0,0,0, 0,0,0,0, 0,0,0,0]
[-1,-1, 1, 1, 0,0,0,0, 0,0,0,0, 0,0,0,0]
[-1,-1,-1,-1, 2,0,0,0, 0,0,2,0, 0,0,0,0]
[-1,-1,-1,-1, 0,2,0,0, 2,0,0,0, 0,0,0,0]
[-1,-1,-1,-1, 0,0,2,0, 0,0,0,2, 0,0,0,0]
[-1,-1,-1,-1, 0,0,0,2, 0,2,0,0, 0,0,0,0]
[-1,-1,-1,-1, 0,0,0,0, 0,0,0,0, 2,2,0,0]
[-1,-1,-1,-1, 0,0,0,0, 0,0,0,0, 0,0,2,2]
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The general solution vector x of the equation system has the 16 components:
$x = (3y_1-y_3-y_2,\ -y_1+y_2+y_3,\ -y_1+y_2+y_3,\ 3y_1-y_3-y_2,\ y_1,\ y_2,\ 2y_1-y_2,\ y_1,\ 2y_1-y_2,\ y_1,\ y_1,\ y_2,$
$-2y_1+2y_2+y_3,\ 4y_1-y_3-2y_2,\ 2y_1-y_3,\ y_3)$, with free variables y_1, y_2, y_3.
Because x has equal components, there is no general 4x4 magic square with this connection figure. (End of example)

For every string from the list, the solution of the corresponding linear equation system is examined, whether its solution has equal components. There are exactly 34 strings, which lead to 16 different components of the solution. When rotations and mirror images are added again, the existence of 145 possible connection figures is proved.